

Electromagnetically Induced Transparency and slow light in media with Rydberg Excitons

Sylvia Zielińska-Raczyńska¹, David Ziemkiewicz¹, Gerard Czajkowski¹

¹ *Institute of Mathematics and Physics, UTP University of Science and Technology,
Al. Prof. S. Kaliskiego 7, PL 85-789 Bydgoszcz (Poland)*

We show that the Electromagnetically Induced Transparency (EIT) is possible in a medium exhibiting Rydberg excitons and indicate the realistic parameters to perform the experiment. The calculations for a Cu_2O crystal are given which show that in this medium due to large group index one could expect slowing down a light pulse by a factor about 10^4 .

I. INTRODUCTION

Recently, a lot of attention has been directed to the subject of excitons in bulk crystals due to experimental observation of the so-called yellow exciton series in Cu_2O up to a large principal quantum number of $n = 25$ [1]-[12]. Such excitons in copper oxide, in analogy to atomic physics, have been named Rydberg excitons. By virtue of their special properties Rydberg excitons are of fascination in solid state and optical physics. These objects, whose size scales as the square of the Rydberg principal quantum number n , are ideally suited for fundamental quantum interrogations, as well as detailed classical analysis. One could expect that Rydberg excitons would have been described, in analogy to Rydberg atoms, by Rydberg series of hydrogen atoms, but it turned out that this generic method of description should have been revised because diameter of such exciton is much larger than wavelength of light needed to create it [5, 12].

The observation and detailed description of Rydberg excitons have opened a new field in condensed matter spectroscopy. In analogy to medium of Rydberg atoms, where it has been possible to obtain a large optical nonlinearity at the single photon level and to realize a lot of quantum optics sophisticated experiments such as optical Kerr effect or correlated states [13], it is expected that the medium of Rydberg excitons is also fertile area. The unique combination of their huge size, long radiative lifetimes, possible strong dipole-dipole interaction and, what is the most important advantage for future technological applications, miniaturization of media/samples they are realized in, can be exploited to perform robust light-exciton quantum interfaces for quantum information processing purposes.

Solid bulk media are systems well worth considering for storing quantum information because they have a number advantages over gases, where a lot of experiments have been done [for recent review see Ref.[13]]: they are easy to prepare, diffusion processes are not so fast, much higher densities of interacting particles can be achieved [14]. A common class of solids used within a quantum information context are rare-earth-metal-doped crystals, where a long time over one minute of storing information has been achieved [15], [17], and nitrogen-vacancy centres in diamond [16] which have relatively long spin coherence. However, the size of solid samples used in such experiments is of order of several millimetres. Rydberg excitonic samples are much smaller: observation of dipolar blockade in bulk Cu_2O [1] and quantum coherence [11] were performed in samples as small as several tens of micrometers. Realization of these experiments have unlocked the plethora of dynamical effect which might be observed in Rydberg excitons media; one of examples is the electromagnetically induced transparency, the performing of which in Cu_2O bulk crystal will be the next step toward potential implementation this medium for quantum information processing.

Electromagnetically induced transparency (EIT) [18], is one of the important effect in quantum optics as it allows for the coherent control of materials' optical properties. The generic EIT bases on extraordinary dispersive properties of an atomic medium with three active states in the Λ configuration. This phenomenon leads to the significant reduction of absorption of a resonant probe, weak laser field by irradiating the medium with a strong control field making an otherwise opaque medium transparent. It leads to dramatic changes of dispersion properties of the system: absorption forms a dip called the transparency window and approaches zero while dispersion in the vicinity of this region becomes normal with a slope, which increases for a decreasing control field. The resonant probe beam is now transmitted almost without losses. EIT has been explained by destructive quantum interference between different excitation pathways of the excited state or alternatively in terms of a dark superposition of states. Since at least 20 years there has been a considered level of activity devoted to EIT [19], which has been motivated by recognition of a number of its applications among which slowing and storing the light, see i.e., [20]-[22] and references therein, which allows for realization of delay lines and buffers in optical circuits are well-known examples. A remarkable

quenching of absorption due to EIT in an undoped bulk of Cu_2O in a Λ -type configuration involving lower² levels was examining in [23]. Demonstration of EIT in Rydberg atoms involving the ladder levels scheme by Mohapatra *et al* [24] has taken advantage of their unique properties. In Rydberg systems the ladder configuration enables to couple long-living metastable, initially empty upper level with the levels coupled by the probe field. Rydberg excitons in Cu_2O offer a great variety of accessible states for creating the ladder configuration what enables such a choice of coupling which could be realized by accessible lasers or eventually to be suitable for desirable coherent interaction implementations. Rydberg EIT has also attracted attention with demonstration of interaction-enhanced absorption imaging of Rydberg excitations [25]. The non-linear response of Rydberg medium is proportional to the group index and to the strength of the dipole-dipole interaction and both of them can be extremely large. EIT provides a possibility of dissipation-free sensing and probing of highly excited Rydberg states and coherent coupling of Rydberg states via EIT could be used for cross-phase modulation and photon entanglement.

Rydberg excitons offer an unprecedented potential to study above mentioned phenomena in solids. It seems that, similar to Rydberg atoms, strong dipolar interaction between Rydberg excitons could appear and leads to so-called photon-blockade, which offers promising means to tailor deterministic single-photon sources [26], and to realize photonic phase gates [27]. Up till now the researches involving in Rydberg excitons, both theoretical and experimental, have concentrated on their static properties (excitonic states, electro- and magneto-optical properties). The first step toward study the photon blockade, quantum non-linear optics, and many-body physics with Rydberg excitons is the realization of Rydberg EIT. Here we focus our interest on dynamical aspects and properties of Rydberg excitons in Cu_2O and show, that the Rydberg excitonic states can be used to perform the EIT. We indicate excitonic states which guarantee of the most efficient realization of the experiment taking into account our previous results concerning excitonic resonances as well as damping parameters and the matrix elements of momentum operator.

Our paper is organized as follows. In Sec. II we present the assumptions of the considered model and solve the time evolution equations, obtaining an analytical expression for the susceptibility and the group index. We use the obtained expression to compute the real and imaginary part of the susceptibility and the group index (Sec. III), for a Cu_2O crystal slab. We examine in details the changes of both real and imaginary part caused by changes of driving parameters (for example, the Rabi frequencies). In Sec. IV we draw conclusions of the model studied in this paper and indicate the optimal choice of Rydberg states to realize the EIT and light slowing. The possible practical importance is also indicated.

II. THEORY

The phenomenon of EIT, described qualitatively above, has been studied theoretically for various configurations of the transitions and probe and control beams. Below we propose a theoretical description for the case when the atomic transitions are replaced by intra excitonic transitions in Rydberg excitons media. Condensed matter exhibits quite a variety of three-level systems where induced transparency could be achieved in much the same way as done with atoms. Yet dephasings, which can easily break the coherence of the population trapping state, are typically much faster in solids than in atomic vapors; it has caused difficulty to observe a large electromagnetic induced transparency effect in solids. This difficulty can be overcome by using the Rydberg exciton states. Higher states have much larger lifetimes and the dephasing can reach the value which enables observation of the EIT effect. Below we will consider a Cu_2O crystal as a medium where the EIT phenomenon can be realized. We use the ladder configuration (Fig. 1), consisting of three levels a, b , and c . As in previous works on Rydberg excitons, we focus our attention on the so-called yellow series associated with the lowest inter-band transition between the Γ_7^+ valence band and the Γ_6^+ conduction band. Because both band-edge states are of even parity, the lowest $1S$ exciton state is dipole-forbidden, whereas all the P states are dipole-allowed; the $1S$ to nP transition is also allowed. We have chosen the valence band as the b state. The $10S$ and $2P$ excitonic states are the a and c states, respectively. To obtain the S state we assume that a constant electric field F is applied to the considered system. As it was recently shown, the applied field splits the P levels into S , P , and D states[12].

Let the probe/signal field of frequency ω_1 and amplitude ε_1 couples the ground state b of energy E_b with an excited state a of energy E_a . The control field of frequency ω_2 and amplitude ε_2 couples a state c of energy E_c with the state a , as it is illustrated in Fig. 1. The Hamiltonian of such three level system interacting

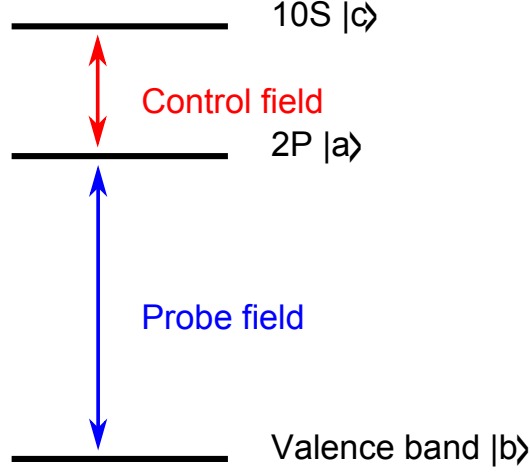


Figure 1: Schematic of the considered ladder EIT system.

with an electromagnetic wave, in the rotating wave approximation reads

$$\begin{aligned}
 H = & E_a|a\rangle\langle a| + E_b|b\rangle\langle b| + E_c|c\rangle\langle c| \\
 & + \{-\hbar\Omega_1(z, t) \exp[-i(\omega_1 t - k_1 z)]|a\rangle\langle b| \\
 & -\hbar\Omega_2(z, t) \exp[-i(\omega_2 t - k_2 z)]|a\rangle\langle c| + h.c.\},
 \end{aligned} \tag{1}$$

where $k_1, k_2, \Omega_1(z, t) = (1/\hbar)d_{ab}\varepsilon_1(z, t)$, $\Omega_2(z, t) = (1/\hbar)d_{ac}\varepsilon_2(z, t)$ are the wave vectors and Rabi frequencies corresponding to the particular couplings and d_{ij} being the dipole transition moments related to the specific transitions. The time evolution of the system is governed by the von Neumann equation with a phenomenological relaxation contribution

$$i\hbar \frac{d\rho}{dt} = [H, \rho] + R\rho, \tag{2}$$

where $\rho(z, t)$ denotes the density matrix for an exciton at the position z , and R is the relaxation operator accounting for all relaxation processes in the medium. Setting

$$\begin{aligned}
 \rho_{ab} &= \sigma_{ab} \exp[-i(\omega_1 t - k_1 z)], \\
 \rho_{ac} &= \sigma_{ac} \exp[-i(\omega_2 t - k_2 z)], \\
 \rho_{bc} &= \sigma_{bc} \exp[-i(\omega_1 - \omega_2)t - (k_1 - k_2)z], \\
 \rho_{aa} &= \sigma_{aa}, \\
 \rho_{bb} &= \sigma_{bb}, \\
 \rho_{cc} &= \sigma_{cc},
 \end{aligned}$$

one can get rid of time-dependent factors except for slowly varying factors accompanying probe field.

We denote by $\delta_1 = (E_a - E_b)/\hbar - \omega_1$ and $\delta_2 = (E_a - E_c)/\hbar - \omega_2$, respectively the probe and the control beam detunings. While propagation effects for the control field are neglected the evolution of the system is

described by the following set of Bloch equations

$$\begin{aligned}
i\dot{\sigma}_{aa} &= -\Omega_1\sigma_{ba} + \Omega_1^*\sigma_{ab} - \Omega_2\sigma_{ca} + \Omega_2^*\sigma_{ac} - i(\Gamma_{ab} - \Gamma_{ca})\sigma_{aa}, \\
i\dot{\sigma}_{bb} &= -\Omega_1^*\sigma_{ab} + \Omega_1\sigma_{ba} + i\Gamma_{ab}\sigma_{aa}, \\
i\dot{\sigma}_{cc} &= -\Omega_2^*\sigma_{ac} + \Omega_2\sigma_{ca} - i\Gamma_{ca}\sigma_{aa}, \\
i\dot{\sigma}_{ab} &= (\delta_1 - i\gamma_{ab})\sigma_{ab} - \Omega_1(\sigma_{bb} - \sigma_{aa}) - \Omega_2\sigma_{cb}, \\
i\dot{\sigma}_{bc} &= (\delta_2 - \delta_1 - i\gamma_{bc})\sigma_{bc} + \Omega_2\sigma_{ba} - \Omega_1^*\sigma_{ac}, \\
i\dot{\sigma}_{ac} &= (\delta_2 - i\gamma_{ac})\sigma_{ca} - \Omega_2(\sigma_{cc} - \sigma_{aa}) - \Omega_1\sigma_{bc}.
\end{aligned} \tag{3}$$

Parameters Γ_{ij} , $i \neq j$, describe damping of exciton states and are determined by exciton-damping mechanisms comprising, temperature-dependent homogeneous broadening due to phonons and broadening due to structural imperfections and eventual impurities. The relaxation damping rates for the coherence are denoted by $\gamma_{ij} \approx \Gamma_{ij}/2$, $i \neq j$ [23]. It should be noted that in the above equations only the relaxations inside the three level system are considered, so the total probability for the populations of the three levels is conserved: $\sigma_{aa} + \sigma_{bb} + \sigma_{cc} = 1$. In order to study propagation of the signal field inside the medium the Bloch equations are accompanied by the Maxwell propagation equation for the Rabi frequency Ω_1 of the probe pulse, which reads in the slowly varying envelope approximation

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \Omega_1 = -i\kappa_1^2 \sigma_{ba}, \tag{4}$$

where $\kappa_1^2 = \frac{N|d_{ab}|^2\omega_1}{2\hbar\epsilon_0}$, N being the density of excitons, d_{ab} is the transition dipole matrix element, ω_1 is the electromagnetic wave frequency.

In the first order perturbation with respect to the probe field, the evolution of our system reduces to the set of the following equations for the density matrix

$$\begin{aligned}
i\dot{\sigma}_{ab} &= (\delta_1 - i\gamma_{ab})\sigma_{ab} - \Omega_1 - \Omega_2\sigma_{cb}, \\
i\dot{\sigma}_{bc} &= (\delta_2 - \delta_1 - i\gamma_{bc})\sigma_{bc} + \Omega_2\sigma_{ba}.
\end{aligned} \tag{5}$$

Taking into account that for a weak probe pulse polarization of the medium for a given frequency is proportional to the signal field ε and to the susceptibility χ it has a form

$$P_1 = \epsilon_0\chi(\omega)\varepsilon_1(\omega) = Nd_{ba}\sigma_{ab}. \tag{7}$$

The steady state complex exciton susceptibility exhibited to the probe field has the form

$$\chi(\omega) = \frac{N|d_{ba}|^2}{\hbar\epsilon_0\Omega_1}\sigma_{ab} = -\frac{N|d_{ab}|^2}{\hbar\epsilon_0} \frac{1}{\omega - \delta_1 + i\gamma_{ab} - \frac{|\Omega_2|^2}{\omega - \delta_1 + \delta_2 + i\gamma_{bc}}}, \tag{8}$$

where ϵ_0 is the vacuum dielectric constant. The susceptibility is a complex, rapidly varying function of ω , its real part describe the dispersion and imaginary show the absorption of the medium. Due to the dependence of the refractive index $n(\omega) = \sqrt{1 + \text{Re}\chi(\omega)}$ on electric susceptibility we define the group index

$$n_g(\omega) = c/v_g = 1 + \frac{\omega_1}{2} \frac{\partial}{\partial \omega} \text{Re}\chi(\omega), \tag{9}$$

which account on time delay of a pulse propagating in a medium with the group velocity

$$v_g = \frac{c}{1 + \frac{1}{2}\text{Re}\chi(0) + \frac{\omega}{2} \frac{\partial \text{Re}\chi(0)}{\partial \omega}},$$

so the slope of dispersion inside the transparency window determines the propagation of the pulse inside the medium. Note that derivative of the real part of the susceptibility may be positive (normal dispersion) or negative (anomalous dispersion); in the latter case the group velocity may even become negative.

Another way of explaining electromagnetically induced transparency applies the notion of dressed states. Consider the subspace spanned by the states a and c (the energy E_c of the latter being moved by the

photon energy $\hbar\omega_2$), coupled by the interaction Ω_2 . The dressed states are eigenvectors of the Hamiltonian restricted to this subspace. The eigenenergies are shifted from their bare values; if the control field is at resonance the shift is equal to $\pm\Omega_2$. If the probe photon is tuned right in the middle between the dressed eigenenergies, the transition amplitudes from the state b interfere destructively.

In the case of spectrally not too wide pulses one can approximate the susceptibility to the lowest term of its Taylor expansion at the line center and in such a case the probe pulse can have the form

$$\epsilon_1(z, t) = \exp\left(i\frac{\omega_1\chi'(0)z}{2c} - \frac{\omega_1\chi''(0)z}{2c}\right)\epsilon_1(0, t - \frac{z}{v_g}). \quad (10)$$

This means that the pulse moves with the velocity v_g with its shape essentially unchanged apart from an exponential modification of its height and an overall phase shift. The group velocity is approximately the velocity of the pulse maximum (exactly if there is no damping).

III. NUMERICAL RESULTS

We have performed numerical calculations for a Cu_2O crystal slab with a thickness $30 \mu\text{m}$. A constant electric field F is applied in the z -direction. Using the formulas (8) and (9) we have calculated the real and imaginary part of the susceptibility and the group index. The values of certain energies, dipole moments and damping parameters characteristic for the excitonic states of Fig. 1 the results from our previous paper [12] have been used. The detailed calculations of $N, |d_{ba}|^2, \gamma_{ab}, \gamma_{bc}\delta_1, \delta_2$ are presented in the Appendix. For N being the density of excitons we have used the value $N = 6.2422 \cdot 10^{19} \text{ cm}^{-3}$. Applying external electric field $F15 \text{ V/cm}$ one obtains the following values of parameters

$$\begin{aligned} \omega_{ab} &= 3266.576 \text{ THz } (=2150.3 \text{ meV}) \\ \omega_{ac} &= 31.402 \text{ THz } (=20.6714 \text{ meV}) \\ \gamma_{ab} &= 45.573 \text{ GHz } (=30 \mu\text{eV}/\hbar) \\ \gamma_{bc} &= 7.596 \text{ GHz } (=5 \mu\text{eV}/\hbar) \\ N &= 6.2422 \cdot 10^{19} \text{ cm}^{-3} \\ M_{01} &= 0.334 \cdot 10^{-60} \end{aligned}$$

As can be seen in Fig. 2 a) for non-zero Rabi frequency of the control field the imaginary part of the system's susceptibility reveals a dip in the Lorentzian absorption profile called a transparency window. This means that a resonant probe beam which otherwise would be strongly absorbed, is now transmitted almost without losses. The width of the transparency window is proportional to the square of control field amplitude and therefore increasing the control field strength it is possible to open it out. The real part of susceptibility is shown in Fig. 2 b). The dispersion inside transparency window becomes normal with the slope which increases from a decreasing control field. The normal dispersion inside the window is responsible for reduction of the group velocity. The absorption at the resonance does not reach zero due to finite value of the relaxation rate γ_{cb} , but it is very small. This means that the medium has become transparent for a probe pulse which travels with a reduced velocity. It is well known that EIT allows us to obtain a steeper slope of the refractive index and thereby a large group index, see Fig. 2 c). The negative values of n_g , corresponding to the regions of anomalous dispersion and high absorption, have been omitted for clarity. The Fig. 2 d) shows how the group index and absorption inside the transparency window depend on the Rabi frequency Ω_2 . The plots represent a cross - sections of Fig. 2 a) and Fig. 2 c) at $\omega = 0$. For $\Omega_2 = 0$, the susceptibility given by Eq. (8) has a single resonance, so that the imaginary part of susceptibility is significant. As the Rabi frequency Ω_2 increases, a transparency window is formed. For considered excitonic transitions optimal slowing down of order 10^{-4} appears for transparency window of width of tens GHz; as shown on the Fig. 2 d), there is some optimum $\Omega_2 \approx 25 \text{ GHz}$ where the group index has a maximum value, corresponding to a narrow, but fully formed transparency window with significant dispersion $\frac{\partial \text{Re}\chi}{\partial \omega}$. By increasing the Ω_2 further, one obtains a wider window, characterized by higher group velocity but also smaller absorption.

It should be stressed that the frequency of the signal to be slowed down or even processed determines the choice of the exciton level involved a . The influence of the damping rate γ_{ab} on the group index and on the

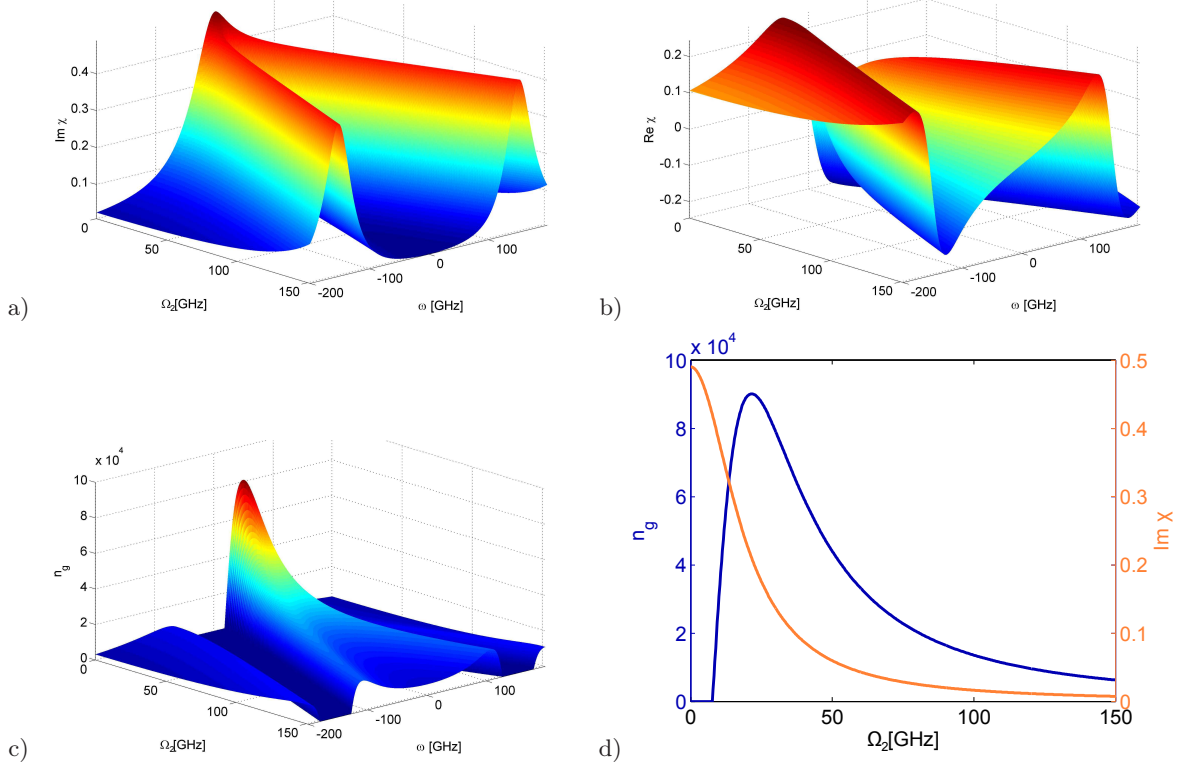


Figure 2: Selected variables as a function of Rabi frequency of the control field Ω_2 . a) Group velocity index $n_g(\omega)$. b) Real part of susceptibility $\chi'(\omega)$. c) Imaginary part of susceptibility $\chi''(\omega)$. d) Group index $n_g(0)$ and $\chi''(0)$, in the center of the transparency window.

group velocity is not crucial. Its increase causes the widening of transparency window which is accompanied by decrease of the dispersion.

IV. CONCLUSIONS

The universal Rydberg nature can be exploited in systems other than atomic gases. The impressive observation of a Rydberg blockade shift on a very different platform offers a new approach for studying semiconductor systems and also provides entirely new long-term perspectives for developing novel devices, which are more robust and compact than atomic systems. We have indicated the optimal states and well justified parameters to attempt the observation EIT in Rydberg excitons Cu_2O media which allow to obtain considerable value of group index. Due to coherence properties of Rydberg excitons the manipulations of the medium transparency is possible; the width of the window and slowing down the group velocity of the pulse travelled inside the sample might be changed in controlled way by the strength of control field. The ability to control on-demand group index enables storing and retrieving light pulses, which is a basis to quantum memory implementation. The way of a precise dynamical control of the optical properties of the medium by optical means reveals new aspects of excitons quantum optics and is supposed to lead to constructing efficient tools for photonics, e.g., delay lines, quantum switches or multiplexers. So far experimental demonstration of EIT in Rydberg excitons media is probably difficult to realize, but it seems safe to expect such experiments in the future. Performing EIT in excitonic Rydberg media will be the step toward realization of controlled interaction of Rydberg excitons in integrated and scalable solid state devices.

For the $2P$ exciton we take the $2Pz$ state, i.e. $|2,1,0\rangle$ state, which couples with the z - component of the electromagnetic wave. So all the waves propagating in the considered medium must have the z - component. According to the notation of Ref. [12] the energy of $2Pz$ exciton follows from the relations

$$W_{200}W_{210} - \left(V_{010}^{(2)}\right)^2 = 0, \quad (\text{A1})$$

with

$$\begin{aligned} W_{n\ell m} &= E_g + E_{n\ell m} - \hbar\omega - i\Gamma = \\ &= E_{Tn\ell m} - E - i\Gamma, \end{aligned}$$

where for $V_{010}^{(n)}$ and in units eFa^* , one obtains

$$\begin{aligned} V_{010}^{(n)} &= \frac{1}{\sqrt{3}} \sqrt{\frac{(n-1)!(n-2)!}{16n!(n+1)!}} \int_0^\infty dx e^{-x} x^4 L_{n-1}^1(x) L_{n-2}^3(x) \\ &= -\sqrt{\frac{12}{n^2(n^2-1)}} \binom{n}{n-2} \binom{n+1}{n-1}. \end{aligned} \quad (\text{A2})$$

The energy eigenvalues have the form

$$E_{n\ell m} = -\frac{\eta_{\ell m}^2}{n^2} R^*, \quad (\text{A3})$$

where $n = 1, 2, \dots, \ell = 0, 1, 2, \dots, n-1, m = -\ell, -\ell+1, \dots, \ell$, with $\eta_{\ell m}$ defined by with

$$\eta_{\ell m} = \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \frac{|Y_{\ell m}|^2}{\sqrt{\sin^2\theta + \gamma^2 \cos^2\theta}}, \quad (\text{A4})$$

$\gamma = \mu_{\parallel}/\mu_z$ being the exciton effective masses anisotropy parameter, L_n^m are the Laguerre polynomials, and $Y_{\ell m}$ being the spherical harmonics. The energy for the $2P$ exciton results from Eq. (A1) where we take the larger value from the two solutions $E_{1,2}$.

The energy for $10S$ exciton could be calculated from the equation

$$W_{10,00}W_{10,10} - \left(V_{010}^{(10)}\right)^2 = 0, \quad (\text{A5})$$

where we take the smaller value from the solutions. The dipole matrix element $|d_{ba}|^2$ follows from the relation

$$|M_{10}|^2 = \frac{4\epsilon_0\epsilon_b a^{*3} \Delta_{LT}^{(P)}}{\pi(r_0/a^*)^2 \eta_{11}^5}, \quad (\text{A6})$$

$\Delta_{LT}^{(P)}$ being the longitudinal-transversal splitting, and r_0 the so-called coherence radius. Positions of our resonances were in perfect agreement with the experimental data (see Fig. 3 a) The damping parameters were determined by fitting our results regarding to the linewidths of resonances. We were able to estimate the dissipation rates of the P excitons (Fig. 3 a) and S excitons (Fig. 3 b) which we later used in our numerical calculations. The relaxation rates are comparable to the values found in prior literature [1].

[1] T. Kazimierczuk, D. Fröhlich, S. Scheel, H. Stolz, and M. Bayer, Nature **514**, 344 (2014).

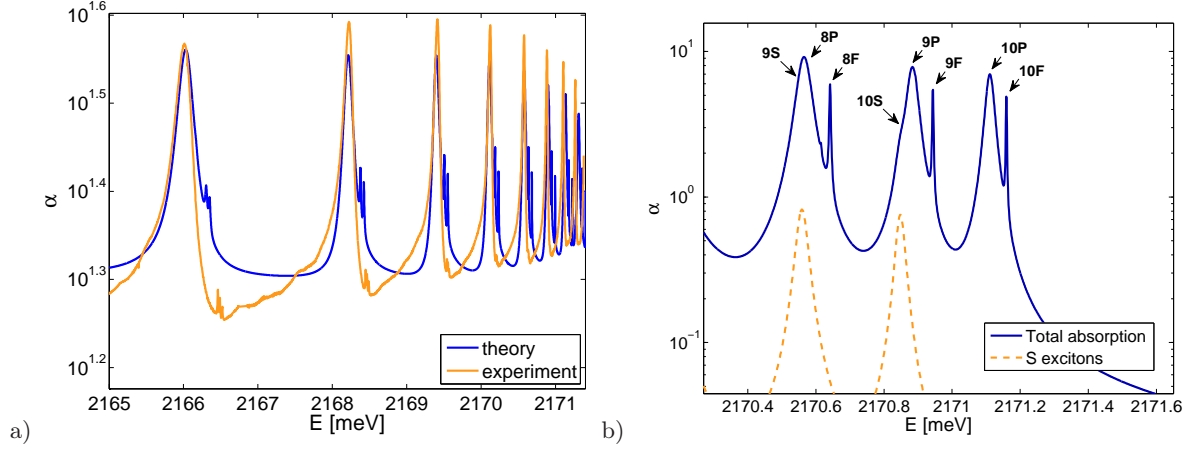


Figure 3: The absorption spectrum of the considered system, for principal quantum number $n = 4 - 10$. a) Comparison of the obtained results with experimental data [28]. The line widths provide the estimation of damping constants γ . b) Detailed spectrum near $n = 10$, showing the overlap of S and P excitonic lines.

- [2] S. Höfling and A. Kavokin, *Nature* **514**, 313 (2014).
- [3] J. Thewes, J. Heckötter, T. Kazimierczuk, M. Aßmann, D. Fröhlich, M. Bayer, M. A. Semina, and M. M. Glazov, *Phys. Rev. Lett.* **115**, 027402 (2015).
- [4] J. Heckötter, *Stark-Effect Measurements on Rydberg Excitons in Cu₂O*. Thesis. Technical University Dortmund, 2015.
- [5] S. Zielińska-Raczyńska, G. Czajkowski, and D. Ziemkiewicz, *Phys. Rev. B* **93**, 075206 (2016).
- [6] F. Schweiner, J. Main, and G. Wunner, *Phys. Rev. B* **93**, 085203 (2016).
- [7] F. Schöne, S.-O. Krüger, P. Grünwald, H. Stolz, M. Aßmann, J. Heckötter, J. Thewes, D. Fröhlich, and M. Bayer, *Phys. Rev. B* **93**, 075203 (2016).
- [8] M. Feldmaier, J. Main, F. Schweiner, H. Cartarius, and G. Wunner, *J. Phys. B: At. Mol. Opt. Phys.* **49**, 144002 (2016).
- [9] F. Schweiner, J. Main, M. Feldmaier, G. Wunner, and Ch. Uihlein, arXiv:1604.00492v1 [cond-mat.mtrl-sci] 2 Apr 2016; *Phys. Rev. B*, accepted (2016).
- [10] M. Aßmann, J. Thewes, D. Fröhlich, and M. Bayer, *Nature Materials*, **15**, 741 (2016).
- [11] P. Grünwald, M. Aßmann, J. Heckötter, D. Fröhlich, M. Bayer, H. Stolz, and S. Scheel, *Phys. Rev. Lett.* **117**, 133003 (2016).
- [12] S. Zielińska-Raczyńska, D. Ziemkiewicz, and G. Czajkowski, *Phys. Rev. B* **94**, 045205 (2016).
- [13] O. Firstenberg, C. S. Adams, and S. Hofferberth, *J. Phys. B: At. Mol. Opt. Phys.* **49**, 152003 (2016).
- [14] M. Johnsson and K. Molmer, *Phys. Rev. A* **70**, 032320 (2004).
- [15] G. Heinze, Ch. Hubrich, and T. Halfmann, *Phys. Rev. Lett.* **111**, 033601 (2013).
- [16] G. D. Fuchs, G. Burkard, P. V. Klimov, and D. D. Awschalom, *Nature Physics* **7**, 789-793 (2011).
- [17] D. Schraft, M. Hain, N. Lorenz, and T. Halfmann, *Phys. Rev. Lett.* **116**, 073602, (2016).
- [18] S. E. Harris, *Phys. Today* **50**, 36 (1997).
- [19] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, *Rev. Mod. Phys.* **77**, 633 (2005).
- [20] A. Racyński, J. Zaremba, and S. Zielińska-Kaniasty, *Phys. Rev. A* **69**, 043801-5 (2004).
- [21] M. Perdian, A. Racyński, J. Zaremba, and S. Zielińska-Kaniasty, *Optics Comm.* **266**, 552-557 (2006).
- [22] A. Racyński, J. Zaremba, and S. Zielińska-Kaniasty, *Phys. Rev. A* **74**, 031810-7 (2007).
- [23] M. Artoni, G. C. La Rocca, and F. Bassani, *Europhys. Lett.* **49**, 445, 2000.
- [24] A. K. Mohapatra, T. R. Jackson, and C. S. Adams, *Phys. Rev. Lett.* **98**, 113030 (2007).
- [25] G. Günter, M. Robert-de-Saint-Vincent, H. Schempp, C. S. Hofmann, S. Whitlock, and M. Weidemüller, *Phys. Rev. Lett.* **108**, 013002 (2012).
- [26] T. Peyronel, O. Firstenberg, Qi-Yu Liang, S. Hofferberth, A. V. Gorshkov, T. Pohl, M. D. Lukin, and V. Vuletić, *Nature* **488**, 57 (2012).
- [27] D. Paredes-Barato and C. S. Adams, *Phys. Rev. Lett.* **112**, 040501 (2014).
- [28] J. Heckötter, private correspondence.